Math 10A
Worksheet, Discussion #29; Thursday, 7/26/2018
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1 Euler's Method

1.1 Concepts

1. Euler's method allows us to approximate solutions to differential equations. Given a differential equation $y' = f(y,t)$ and an initial condition $y(0) = y_0$ and a step size h, we can approximate the path by $y_{n+1} = y_n + f(y_n, t_n)h$. This is gotten by writing $y' = \frac{dy}{dt} \approx \frac{y_{n+1} - y_n}{h}$.

   A slope field is a graph where at every point y, t, you draw a line with the slope there, which is given by the function $f(y,t)$.

1.2 Problems

2. True False We can only use slope fields and Euler's method when we are given a first order equation.
3. Consider the differential equation $y' = x - y^2$ with initial condition $y(0) = 1$. Use Euler's method to approximate $y(3)$ using step sizes of 1.
4. Use Euler's method to estimate $y(3)$ given that $y' = x^2 + y^2$ and $y(0) = 0$ using step sizes of 1.
5. Use Euler's method to estimate $y(3)$ given that $y' = y^2 - x^2$ and $y(0) = 1$ using step sizes of 1.

2 Slope Fields

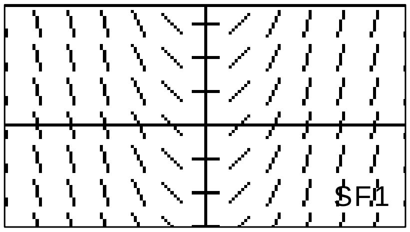
2.1 Concepts

6. A slope field is a graph where at every point y, t, you draw a line with the slope there, which is given by the function $f(y,t)$.

2.2 Problems

7. True False Autonomous equations like $y' = 2\sqrt{y}$ will have slope field that are the same after shifting left and right.

8. Match each slope field to the differential equation and sketch some solutions to them.
9. Draw a slope field for $y' = y^2 + x^2$ and sketch the solution when $y(0) = 0$ on the interval $-2 \le x \le 2, -2 \le y \le 2$.
10. Draw a slope field for $y' = y^2 - x^2$ and sketch the solution when $y(0) = 1$ on the interval $0 \le x \le 4, 0 \le y \le 4$.
11. For each differential equation, estimate $y(2)$ using the starting point $y(1) = 1$ and step size of $h = \frac{1}{2}$.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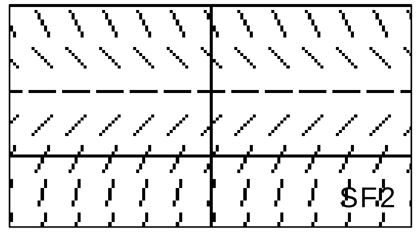
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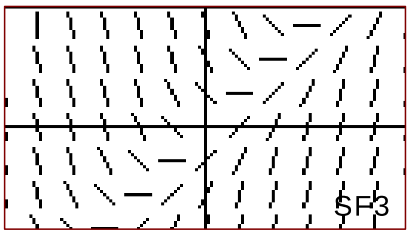
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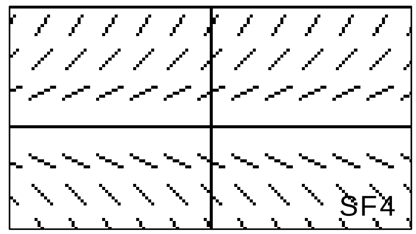
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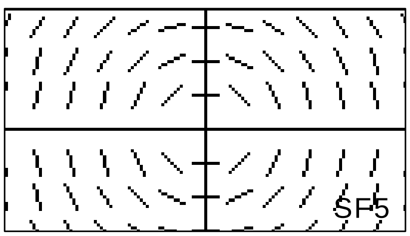
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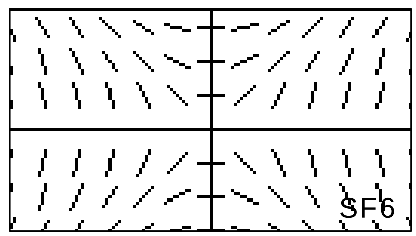
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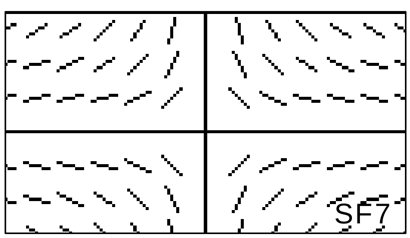
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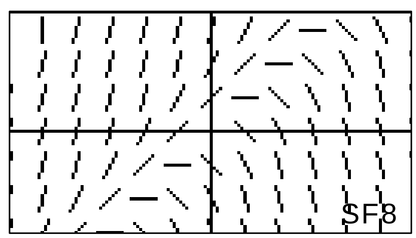
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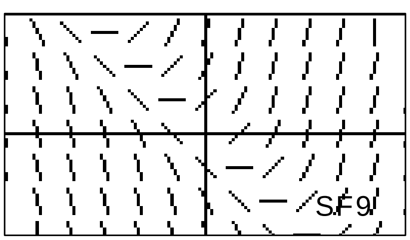
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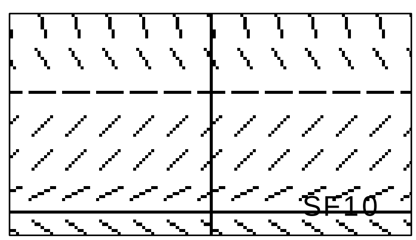
SF10

$$\frac{dy}{dx} = x - y$$

DE1

$$\frac{dy}{dx} = \frac{x}{y}$$

DE2

$$\frac{dy}{dx} = y - x$$

DE3

$$\frac{dy}{dx} = -\frac{x}{y}$$

DE4

$$\frac{dy}{dx} = x$$

DE5

$$\frac{dy}{dx} = -\frac{y}{x}$$

DE6

$$\frac{dy}{dx} = \frac{y}{2}$$

DE7

$$\frac{dy}{dx} = 0.25y(4 - y)$$

DE8

$$\frac{dy}{dx} = 2 - y$$

DE9

$$\frac{dy}{dx} = x + y$$

DE10